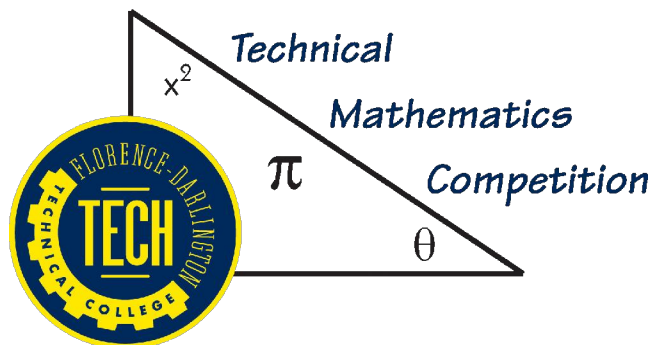


Florence-Darlington Technical College Technical Mathematics Contest Sample Problems

Problem Set 1



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Enclosed are suggested problems for review for participation in the 2006 math contest. The problems are similar to problems used in previous contests. All problems use concepts and skills from arithmetic, geometry, algebra, and trigonometry. Students will be allowed to use calculators on all problems. Florence-Darlington Technical College requires either a TI-83 or TI-84 series calculator for all mathematics courses.

(All answers must include correct units and where necessary be rounded as requested. All problems should be completed by rounding only the final answer.)

1. Susie drove the 232 miles to her grandmother's house in 4 hours and 15 minutes. On the return trip, there was construction on the highway, so she took local roads. The return trip took 6 hours and 30 minutes to drive 283 miles. What was Susie's average speed for the trip? (If necessary, round your answer to the nearest 10th.) - *Suggested Time Limit: 3 minutes*

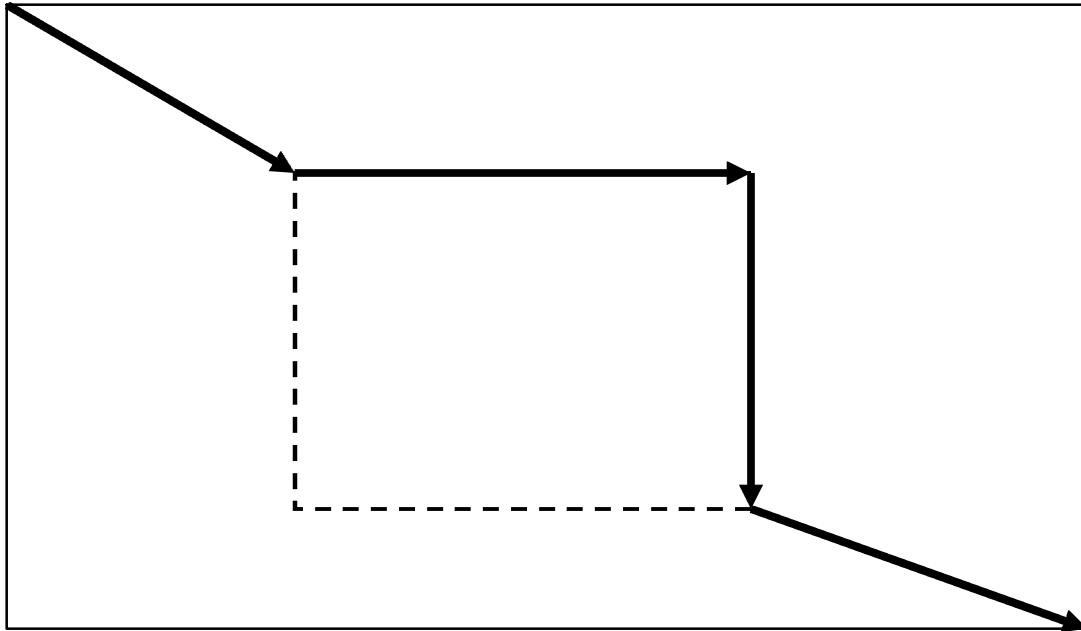
$$\frac{232 \text{ mi.} + 283 \text{ mi.}}{4 \text{ hr. } 15 \text{ min.} + 6 \text{ hr. } 30 \text{ min.}} = \frac{515 \text{ mi.}}{4\frac{1}{4} \text{ hr.} + 6\frac{1}{2} \text{ hr.}} = \frac{515 \text{ mi.}}{10.5 \text{ hr.}} = 49.0 \text{ mph}$$

2. A person's basal metabolic rate (BMR) is the rate at which your body burns calories while you're at rest. Use the formula below to calculate the BMR of 34-year-old Kenya who lives in France. Kenya is 173 cm tall and weighs 65.9 kg. (Use 1 in. = 2.54 cm and 1 kg = 2.2 lbs to make unit conversions. (If necessary, round your answer to the nearest whole number.)

BMR = 655 + (4.3 x weight in pounds) + (4.7 x height in inches) - (4.7 x age in years)
- *Suggested Time Limit: 5 minutes*

$$\text{BMR} = 655 + 4.3 \times \frac{65.9 \cancel{\text{ kg}} \times 2.2 \text{ lb.}}{1 \cancel{\text{ kg}}} + 4.7 \times \frac{173 \cancel{\text{ cm}} \times 1 \text{ in.}}{2.54 \cancel{\text{ cm}}} - 4.7 \times 34 \text{ yrs.} = 1438.7 \text{ Cal.}$$

3. As Stephen walks home he takes a short cut through a lot with a vacant building as shown by the dotted lines. His path is shown by the bold rays. The dimensions of the lot are 462 feet by 287 feet. The north side of the vacant building is 132 feet. The width of the building is 78 feet. If the distance from the north boundary to the building is 80 feet and the distance from the west boundary to the building is 120 feet, to the nearest tenth of a foot, how many feet shorter is the path he chose compared to walking the length and width of the lot? (Note: The top of the drawing is north. The drawing is not drawn to scale.)



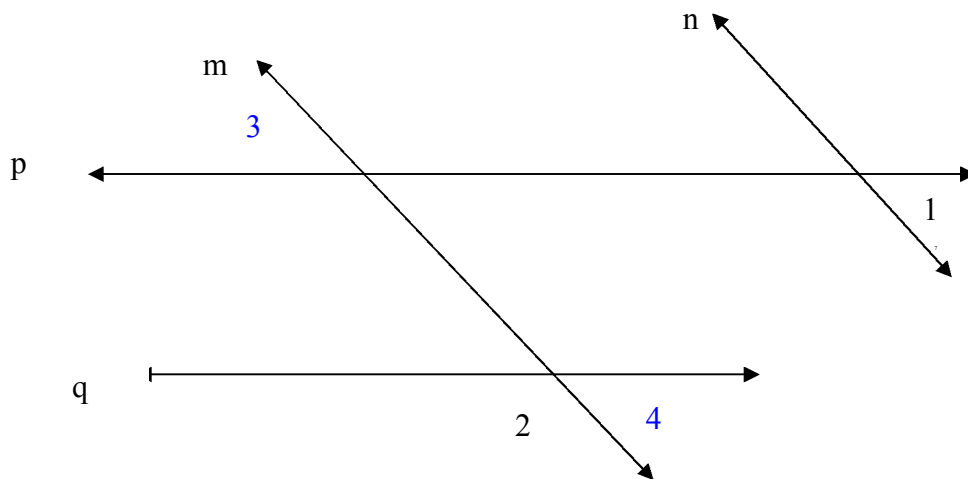
- Suggested Time Limit: 5 minutes

$$\text{Street Path} = (462 + 287) \text{ ft.} = 749 \text{ ft.}$$

$$\text{Short Cut} = \left(\sqrt{120^2 + 80^2} + 132 + 78 + \sqrt{(287 - 80 - 78)^2 + (462 - 120 - 132)^2} \right) \text{ ft.} = 600.7 \text{ ft.}$$

$$\text{Difference} = (749 - 600.7) \text{ ft.} = 148.3 \text{ ft.}$$

4. Lines m and n are parallel lines, and p and q are parallel lines. If $m\angle 1 = 47^\circ 36' 21''$, find $m\angle 2$. (Give your answer in degrees, minutes, and seconds).



- *Suggested Time Limit: 3 minutes*

$m\angle 1 = m\angle 3 = m\angle 4$, $\angle 2$ and $\angle 4$ are supplementary.

Therefore $m\angle 2 = 180^\circ - 47^\circ 36' 21'' = 132^\circ 23' 39''$

5. Imagine that a rich uncle died and as his only heir, he left you all of his money. You uncle was eccentric and loved practical (and sometimes not so practical) jokes. Before his death, he converted all of his assets to cash. He asked for all of it in \$2 bills and placed them in a bank lock box. His will stipulated that you must count all of the money and give the correct total before it becomes yours. Further, you are required to count it in the bank vault. The bank is open from 9-5, Monday through Friday. You are allowed to take 30 minutes for lunch and two 10-minute breaks, one before lunch and one after lunch each day. You count at the rate of 1 bill per second. You stay at the bank from opening until closing each day and utilize the maximum available amount of time each day. You successfully count the money on your first attempt. It takes you 17 weeks, 2 days, and 4 hours to count the money. How much money did your uncle leave you? - *Suggested Time Limit: 5 minutes*

1 work day = 8 hr. – 30 min. – 10 min. – 10 min. = 430 min.

17 wks. = $430 \frac{\text{min.}}{\text{day}} \times 5 \frac{\text{days}}{\text{wk.}} \times 17 \text{wks.} = 36,550 \text{ min.}$

2 days = $430 \frac{\text{min.}}{\text{day}} \times 2 \text{ days} = 860 \text{ min.}$

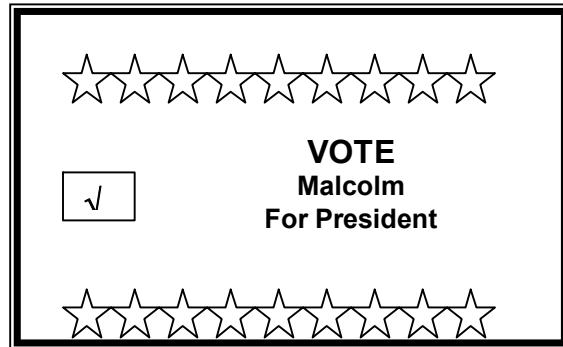
4 hrs. = $60 \frac{\text{min.}}{\text{hr.}} \times 4 \text{ hrs.} = 240 \text{ min.}$

17 weeks, 2 days, and 4 hours

= 36,550 min. + 860 min. + 240 min. = 37,650 min.

Total Inheritance = $37,650 \text{ min.} \times 60 \frac{\text{sec.}}{\text{min.}} \times \frac{\$2}{\text{sec.}} = \$4,518,000$

6. Malcolm is running for president of the student body association. He orders a poster 4 feet wide and 2 feet high. Rita, Malcolm's campaign manager, decides the poster looks out of proportion and estimates that it needs to be increased by 1008 square inches to make it look just right. If each dimension of the poster is increased by the same amount, what will be the dimensions of the new poster? (Express the dimensions in inches.)



- Suggested Time Limit: 3 minutes

Original poster is 48 in. \times 24 in. = 1152 in.²

New poster is $(x+48)$ in. \times $(x+24)$ in.

$$(x+48)(x+24) = 1152 + 1008$$

$$x^2 + 72x + 1152 = 2160$$

$$x^2 + 72x - 1008 = 0$$

$$(x+84)(x-12) = 0$$

$$x + 84 = 0 \text{ or } x - 12 = 0$$

$$x = -84 \text{ or } x = 12$$

-84 is extraneous; 12 in. is the only possible solution.

12 in. was added to each dimension.

7. An airplane flying at an altitude of 32,000 feet descends at a rate of 1,280 feet per minute. Write a linear function to find the altitude at any time t . Find the plane's altitude after 10 minutes and 15 seconds. Round all values to the nearest foot.

- Suggested Time Limit: 3 minutes

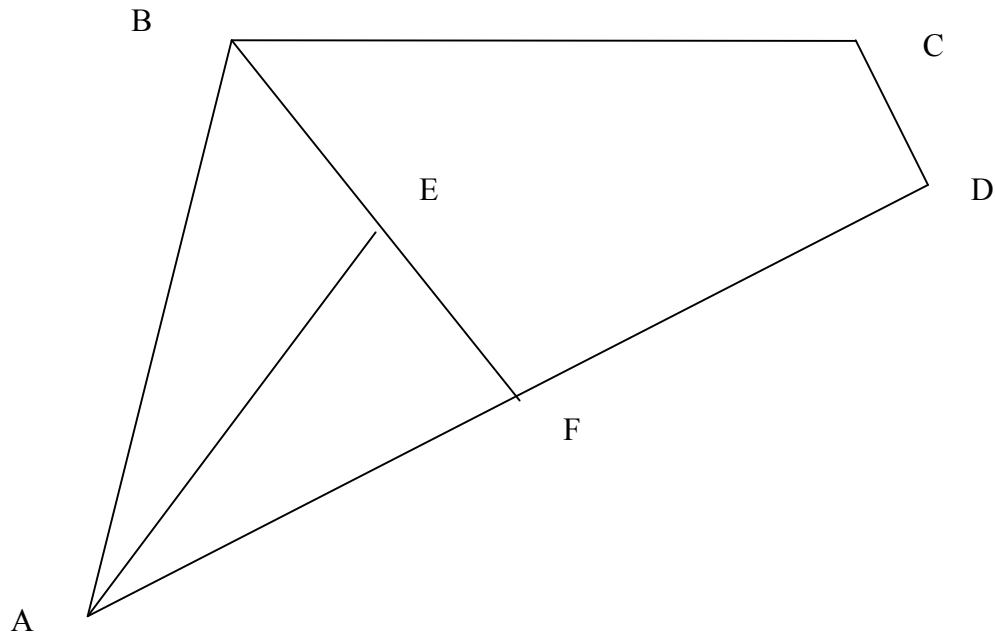
1) The altitude as a function of time is: $a(t) = 32,000 - 1,280t$

2) The plane's altitude after 10 min. and 15 sec. gives $t = 10.25$ sec.

$$a(10.25) = 32,000 - 1,280(10.25) = 18,880 \text{ ft.}$$

8. In the figure below, E is the midpoint of segment BF and F is the midpoint of segment AD. E has coordinates (6, 3), D has coordinates (21, 4), and A has coordinates (-3, -6). To the nearest 10th square unit, what is the area of $\triangle ABF$? (Hint: Use Heron's formula.)

- Suggested Time Limit: 5 minutes



(1) First use the midpoint formula to find the coordinates of point F.

$$F\left(\frac{-3+21}{2}, \frac{-6+4}{2}\right) = F(9, -1).$$

(2) E is the midpoint of \overline{BF} , therefore the coordinates of B are (3, 7).

(3) Use the distance formula to find the lengths of the sides of the triangle.

$$AB = m\overline{AB}.$$

$$\text{Let } a = AB = \sqrt{(3 - (-3))^2 + (7 - (-6))^2} = 14.32$$

$$b = AF = \sqrt{(9 - (-3))^2 + (-1 - (-6))^2} = 13$$

$$c = BF = \sqrt{(9 - 3)^2 + (-1 - 7)^2} = 10$$

(4) Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\text{where } s = \frac{a+b+c}{2} = \frac{14.32+13+10}{2} = 18.66$$

$$A = \sqrt{18.66(18.66 - 14.32)(18.66 - 13)(18.66 - 10)} = 63.0 \text{ units}^2.$$

9. On a certain test, the average score for women is 83. The average score for men is 71. The overall average was 80. What percent of the class were women? - *Suggested Time Limit: 3 minutes*

Let x = percent of women and x = percent; we therefore can form and solve the following system of equations and solve for x only.

$$71x + 83y = 80(x + y)$$

$$x + y = 100$$

Solving for x gives: $x = 75\%$ women

10. A fuel storage tank is a right circular cylinder with a base of diameter 2.35 meters and a height of 4.1 meters. If 1 meter = 39.37 inches and one cubic foot of water = 7.5 gallon. To the nearest gallon, what is the capacity of the tank? (Note – Use the pi key on your calculator and round only the final answer.) - *Suggested Time Limit: 5 minutes*

$$l = 2w$$

$$A = lw = (2w)(w) = 3w^2$$

$$3w^2 =$$

$$V = \pi r^2 h = \pi \left(\frac{2.35 \text{ m}}{2} \right)^2 (4.1 \text{ m}) = 5.0660525\pi \text{ m}^3 \times \left(\frac{39.37 \text{ in.}}{1 \text{ m}} \right)^3 \times \left(\frac{1 \text{ ft.}}{12 \text{ in.}} \right)^3 \times \left(\frac{7.5 \text{ gal.}}{1 \text{ ft.}^3} \right)$$

$$V = 4,710.0 \text{ gal.}$$

11. A supply duct is to run the perimeter of a building having a length equal to twice the width. The area of the building is 41,472 square feet. Find the length of the duct? - *Suggested Time Limit: 3 minutes*

$$l = 2w$$

$$A = lw = (2w)(w) = 2w^2$$

$$2w^2 = 41,472$$

$$w^2 = 20,736$$

$$w = 144 \text{ ft.}, l = 288 \text{ ft.}$$

$$P = 2l + 2w = 2(288) + 2(144) = 864 \text{ ft.}$$

12. A bungee jumper always rebounds 60% of the distance fallen. A bungee jump is made using a cord that stretches to 200 feet. How many feet will the jumper travel in 5 complete rebounds? (If necessary, round your final answer

to the nearest foot.) - *Suggested Time Limit: 5 minutes*

Total Distance =

$$\begin{aligned} 200 + .6(200) + .6(200) + .6^2(200) + .6^2(200) + .6^3(200) + .6^3(200) + .6^4(200) + .6^4(200) + .6^5(200) \\ = 200 \left[1 + 2(.6) + 2(.6)^2 + 2(.6)^3 + 2(.6)^4 + .6^5 \right] \\ = 737.8 \text{ ft.} \end{aligned}$$

13. If 68% of the adult population of a village is registered to vote, and 55% of those registered actually voted, to the nearest 10th percent what percent of adults in the village did NOT vote in the election? - *Suggested Time Limit: 3 minutes*

$$\text{Percent not voting} = 100\% - (55\%)(68\%) = 62.6\%$$

14. Coyotes are one of the few species of North American animals with an expanding range. The future population of coyotes in hundreds in a region of Mississippi can be modeled by the equation $P = 80 + 12 \ln(19t + 1)$, where t is the time in years. Use the equation to determine when the population will reach 17,000. (If necessary, round your answer to the nearest tenth of a year.) - *Suggested Time Limit: 5 minutes*

$$17,000 = \frac{17,000}{100} = 170 \text{ hundreds}$$

$$80 + 12 \ln(19t + 1) = 170$$

$$12 \ln(19t + 1) = 90$$

$$\ln(19t + 1) = 7.5$$

$$19t + 1 = e^{7.5}$$

$$19t = e^{7.5} - 1$$

$$t = \frac{e^{7.5} - 1}{19}$$

$$t = 95.1 \text{ yrs.}$$

15. The long cable between the ends of a suspension bridge is 50 ft above the roadbed at the ends of the bridge and 10 ft above it in the center of the bridge. The path of the cable is a parabola. The roadbed is 200 ft long. Find the length of a vertical cable 20 feet from the center of the bridge. (Hint: Sketch a model using the coordinate plane. Use a system of equations.) If necessary, round your answer to the nearest tenth of a foot.) - *Suggested Time Limit: 5 minutes*

(insert drawing)

From the drawing we have three points on the curve of the parabola. Substitute the coordinates of the three points for x and y into the general form of the quadratic function. You will have a system of three linear equations which may be solved for the coefficients of a , b , and c .

General form of quadratic function: $y = ax^2 + bx + c$

$$\left. \begin{array}{l} 50 = a(-100)^2 + b(-100) + c \\ 10 = a(0)^2 + b(0) + c \\ 50 = a(100)^2 + b(100) + c \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} 10,000a - 100b + c = 50 \\ c = 10 \\ 10,000a + 100b + c = 50 \end{array} \right.$$

Solving the above system: $a = 0.004$, $b = 0$, $c = 10$

Therefore the equation for the cable is: $y = 0.004x^2 + 10$ or $f(x) = 0.004x^2 + 10$

The length of the vertical cable 20 feet from the center will equal the value $f(20)$.
 $f(20) = 0.004(20)^2 + 10 = 11.6$ ft.

16. A wind has an easterly component of 18 mph and a southerly component of 13 mph. Find the magnitude and direction of the wind. If necessary, round each to the nearest tenth unit. Use miles per hour for the magnitude and notation like $N20^\circ W$ for the direction. - *Suggested Time Limit: 5 minutes*

See the drawing below.

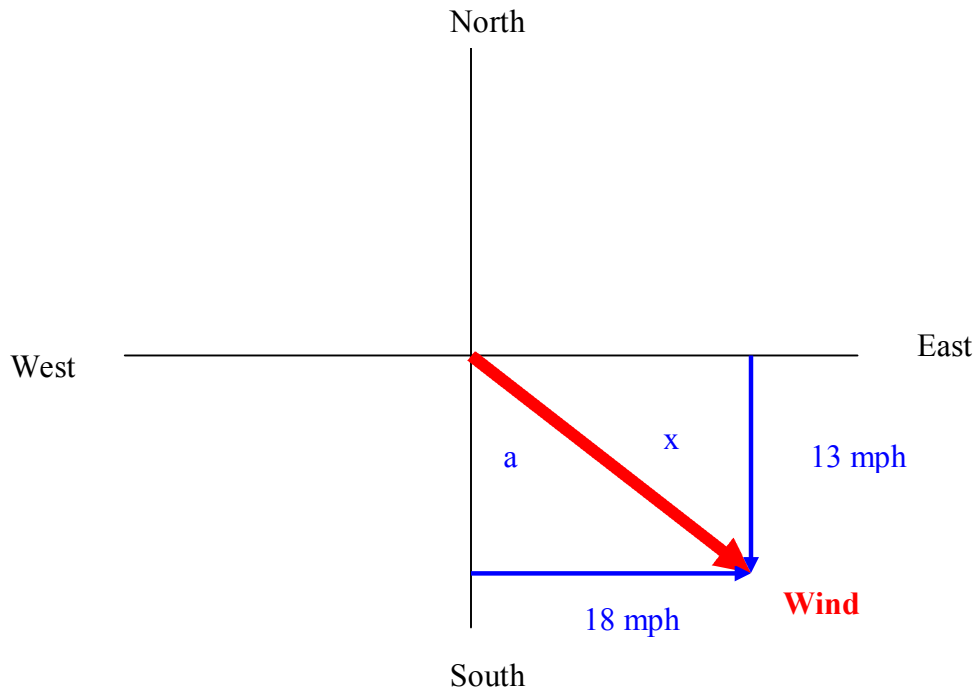
(1) The magnitude: $x = \sqrt{18^2 + 13^2} = 22.2$ mph

$$\tan(\angle a) = \frac{18}{13} = 1.3846$$

$$m\angle a = \tan^{-1}(1.3846)$$

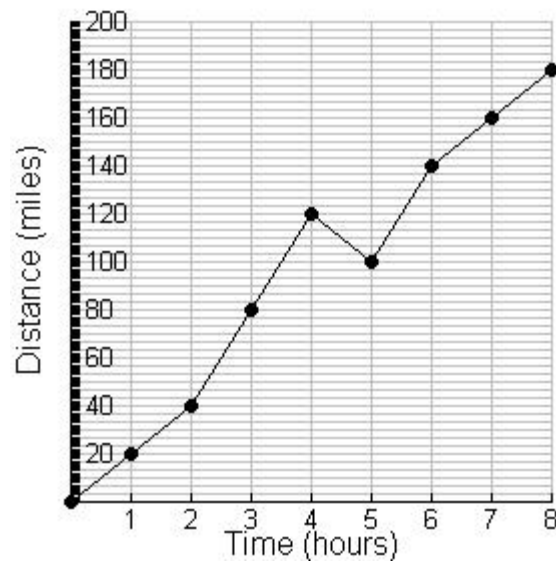
(2) The direction: $m\angle a = 54.2^\circ$

The wind has a direction of $S 54.2^\circ E$ and a magnitude of 22.2 mph.



17. Use the distance vs. time graph and data table below to determine the average velocity for the entire 8 hours.

time	distance
0	0
1	20
2	40
3	80
4	120
5	100
6	140
7	160
8	180



- Suggested Time Limit: 3 minutes

$$\text{Avg. Velocity} = \frac{d_f - d_i}{t_f - t_i} = \frac{180 - 0}{8 - 0} = \frac{180}{8} = 22.5 \text{ mph}$$

18. You are a forensic lab technician. You have been asked to identify a shard of metal found at a crime scene. You know that one way to analyze metal is to find its density using the formula:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

You find the mass of the shard to be 35.2 g. By displacement, you determine the volume of the shard to be 4.5 milliliters. The table below gives the densities of some metals. What type of metal does your shard most likely consist of?

Metal	Density (g/ml)
Magnesium	1.7
Aluminum	2.7
Iron	7.8
Cooper	8.7
Silver	10.5
Lead	11.3

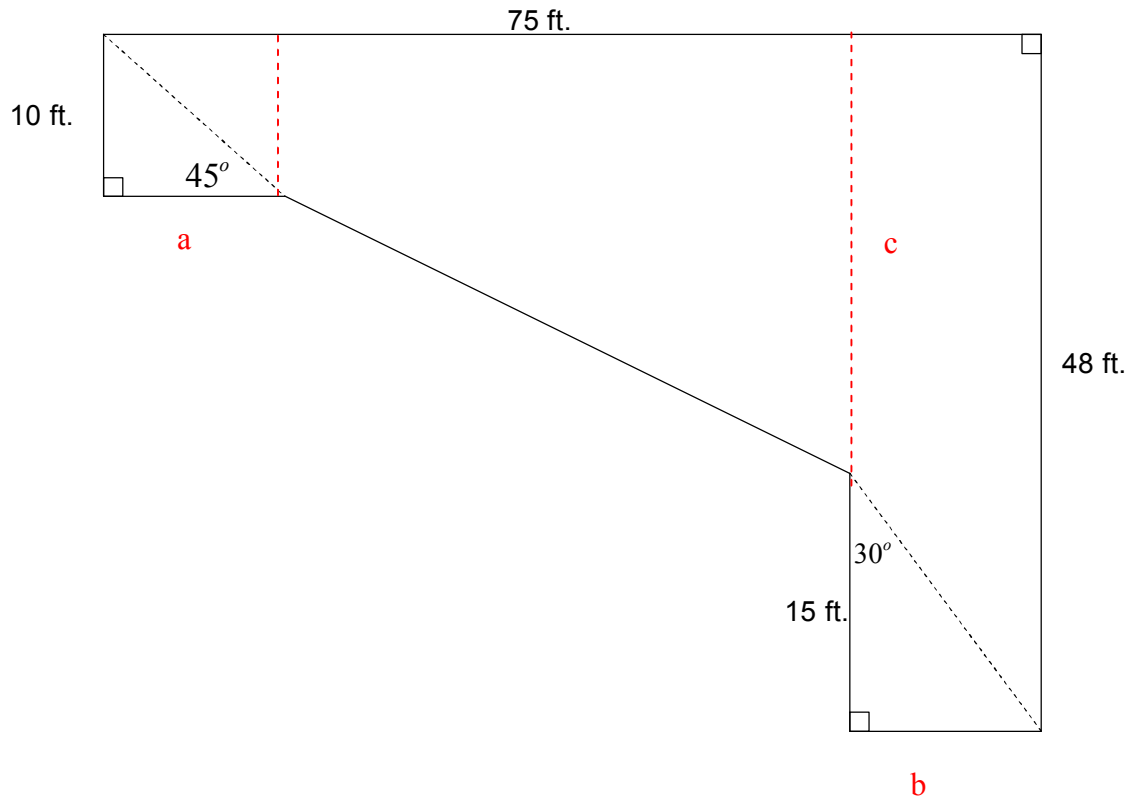
- Suggested Time Limit: 3 minutes

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{35.2 \text{ g}}{4.5 \text{ ml}} = 7.82 \text{ g/ml}$$

The shard is probably made of iron.

19. Find the area of the following. Round your answer to the nearest square foot.

- Suggested Time Limit: 3 minutes



Divide the figure into the three simple figures shown above: a square, trapezoid, and a rectangle. Find the lengths of segments a, b, and c. Then find the area of the respective figures and combine their areas.

$$a = 10 \text{ ft.}, b = \frac{15}{\sqrt{3}} = 8.66 \text{ ft.}, c = 48 - 15 = 33 \text{ ft.}$$

$$\text{Area of the square: } A_s = s^2 = 10^2 = 100 \text{ ft.}^2$$

$$\text{Area of the trapezoid: } A_t = h(b_1 + b_2) / 2 = \frac{(75 - 10 - 8.66)(33 + 10)}{2} = 1211.3 \text{ ft.}^2$$

$$\text{Area of the rectangle: } A_r = lw = 48 \times 8.66 = 415.7 \text{ ft.}^2$$

$$\text{Area of figure: } A_T = 100 + 1,211.3 + 415.7 = 1,727 \text{ ft.}^2$$

20. At Moscatello's Italian Restaurant, pizza comes in two sizes.
 14-in. square
 16-in. round

Which is the larger pizza and by how much? Give your answer to 2 decimal places. - *Suggested Time Limit: 3 minutes*

Square pizza: $A_s = s^2 = 14^2 = 196 \text{ in.}^2$

Round pizza: $A_c = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{16}{2}\right)^2 = 201.06 \text{ in.}^2$

Difference: $A_c - A_s = 201.06 - 196 = 5.06 \text{ in.}^2$

The 16 in.-round is larger by 5.06 in.^2 .